Foundation for Success

Unified International
Mathematics Olympiad

## UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD

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CLASS - }
    Question Paper Code : UM9274
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| $\mathrm{~A}, \mathrm{~B}, \mathrm{C}$ | $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ | $\mathrm{A}, \mathrm{B}, \mathrm{D}$ | $\mathrm{A}, \mathrm{B}, \mathrm{C}$ | $\mathrm{A}, \mathrm{B}, \mathrm{D}$ | C | C | C | A | C |
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## EXPLANATIONS

## MATHEMATICS - 1 (MCQ)

1. (B) $\mathrm{CP}=\mathrm{CP}$ of 40 mts thread
$\mathrm{P}=\mathrm{CP}$ of 8 mts thread
Profit\% =
$\frac{P}{C P} \times 100=\frac{C P \text { of } 8 \mathrm{mts} \text { thread }}{\mathrm{CP} \text { of } 40 \mathrm{mts} \text { thread }} \times 100=20 \%$
2. (D) Remaining part $=1-\left(\frac{1}{3}+\frac{1}{6}\right)=\frac{1}{2}$

Average rate \% per annum (R)

$$
\begin{aligned}
& =\left(\frac{1}{3} \times 3\right)+\left(\frac{1}{6} \times 6\right)+\left(\frac{1}{2} \times 8\right)=6 \% \\
& S . I=₹ 600 \\
& T=2 \text { years, } P=? \\
& I=\frac{P T R}{100} \\
& \Rightarrow P=\frac{100 \times I}{T R}=\frac{100 \times 600}{2 \times 6} \\
& =₹ 5000
\end{aligned}
$$

3. (C) $x y-x-y+1=1$
$y(x-1)-1(x-1)=1$
$(y-1)(x-1)=1$
as $x$ and $y$ are integers, $x-1$ and $y-1$ are integers.

Hence, $x-1$ and $y-1$ must both be 1 or -1 .
i.e, $x=2, y=2$ or $x=0, y=0$

Hence only 2 integer pairs satisfy the condition $x+y=x y$
Note: $2+2=2 \times 2$ and $0+0=0 \times 0$,
remember $0 \times 0=0$ but $0 \div 0$ is not defined.
4. (C) $A B=6 \mathrm{~m}, \mathrm{CD}=11 \mathrm{~m}, \mathrm{AC}=12 \mathrm{~m}$

Now, $D E=(C D-C E)=(11-6) m=5 \mathrm{~m}$


In $\triangle \mathrm{BED}$,
$\mathrm{BD}^{2}=\mathrm{BE}^{2}+\mathrm{DE}^{2}$
$=(12)^{2}+(5)^{2}$
$=144+25=169$
$\therefore \quad B D=13 \mathrm{~m}$
$\therefore \quad$ Answer is option (C).
5. (C) Distance covered in 1 revolution $=$
$\frac{88 \times 1000}{1000} \mathrm{~m}=88 \mathrm{~m}$
$2 \pi R=8$
$\Rightarrow 2 \times \frac{22}{7} \times R=88$
$\Rightarrow R=\left(88 \times \frac{7}{44}\right)=14$
$\therefore \quad$ Diameter $=2 R=(2 \times 14) \mathrm{m}=28 \mathrm{~m}$
6. (D) The arithmetic mean (average) of $3^{10}, 3^{20}$ and $3^{30}$ is
$\frac{3^{10}+3^{20}+3^{30}}{3}=\frac{3^{10}}{3}+\frac{3^{20}}{3}+\frac{3^{30}}{3}$
$=3^{9}+3^{19}+3^{29}$
7. (D) Given $\angle \mathrm{AOD}+\angle \mathrm{DOB}=180^{\circ}$
$2 x+3 x=180^{\circ}$
$x=36^{\circ}$
Given $\angle \mathrm{DOB}=3 x$
$\Rightarrow \angle \mathrm{DOG}+\angle \mathrm{EOB}=3 x$
$2 z+z=3 x$
$3 z=3 x$
$\therefore \quad z=x$
$\therefore \quad x=z=36^{\circ}$


Given $\angle \mathrm{AOD}=2 x$
$\Rightarrow \angle \mathrm{AOC}+\angle \mathrm{COD}=2 x$.
$\frac{y}{4}+\frac{3 y}{4}=2 x$
$y=2 x$
$\therefore \angle \mathrm{COE}=\angle \mathrm{COD}+\angle \mathrm{DOE}=\frac{3 y}{4}+2 z=\frac{3}{4}(2 x)+2 x$
$=\frac{3 x}{2}+2 x=\frac{7 x}{2}$
$=\frac{7 \times 36^{\circ}}{2}=126^{\circ}$
8. (B) $\frac{25}{19}=1+\frac{6}{19}$
$=1+\frac{1}{\left(\frac{19}{6}\right)}$
$\therefore P+\frac{1}{q+\frac{1}{r}}=1+\frac{1}{3+\frac{1}{6}}$
$\therefore r=6$
9. (B) Cost of one litre milk

$$
=\frac{\left(₹ \frac{5157}{20}\right)}{\frac{27}{5}}=\frac{₹ 5157}{20} \times \frac{5}{27}
$$

$\therefore \quad$ Cost of $3 \frac{3}{4}$ litres

$$
=\frac{₹ 515 \xi^{191}}{4} \times \frac{1}{27} \times \frac{15}{4}=\frac{₹ 2865}{16}=₹ 179 \frac{1}{16}
$$

10. (B)

$$
\begin{aligned}
& \text { Mean }=\frac{25+26++25+27+30+28+31+27+33+27+29}{11} \\
&=\frac{308}{11}=28 \\
& \text { Mode }=27
\end{aligned}
$$

$$
\text { Mean }- \text { Mode }=28-27=1
$$

11. (D) LHS $=\frac{5}{8}$ of $\frac{24}{25} \div\left[\frac{5}{3}+\frac{1}{2}\left\{\frac{3}{4} \div\left(\frac{8}{5} \times \frac{5}{2}\right)-\frac{4}{3}\right\}\right]$

$$
\begin{aligned}
& =\frac{3}{5} \div\left[\frac{5}{3}+\frac{1}{2}\left\{\frac{3}{4} \times \frac{1}{4}-\frac{4}{3}\right\}\right] \\
& =\frac{3}{5} \div\left[\frac{5}{3}+\frac{1}{2}\left\{\frac{9-64}{48}\right\}\right] \\
& =\frac{3}{5} \div\left[\frac{5}{3}-\frac{55}{96}\right] \\
& =\frac{3}{5} \div\left[\frac{160-55}{96}\right] \\
& =\frac{3}{5} \times \frac{96}{105} \\
& =\frac{96}{175}
\end{aligned}
$$

12. (C) $(-1)^{2023}+(-1)^{2024}=-1+1=0$
13. (D) Given $4 x-7=2 x+5$
$2 x=12$
$x=6$
$\therefore \quad 2 x+5=2(6)+5=12+5=17 \mathrm{~cm}$
14. (D) Given $x+3 x=90^{\circ}$
$4 x=90^{\circ}$
$x=\frac{90^{\circ}}{4}=22.5$
$3 x=67.5^{\circ}$
15. (D) $4^{\left(\frac{4 x+1}{2}\right)^{\frac{1}{3}}}=2^{-5}$

$$
\begin{aligned}
& 2^{2\left(\frac{4 x+1}{6}\right)}=2^{-5} \\
& \frac{4 x+1}{3}=-5 \\
& 4 x+1=-15 \\
& 4 x=-16 \\
& x=-4
\end{aligned}
$$

16. (A) $\triangle P Q R$ is equilateral.
$\therefore \quad$ Each of its angles is $60^{\circ}$
$\therefore \quad x=60^{\circ}+60^{\circ}=120^{\circ}$
In $\triangle \mathrm{PRS}, \mathrm{PR}=\mathrm{RS}$
$\Rightarrow x+y+y=180^{\circ}$
$120^{\circ}+2 y=180^{\circ}$
$\Rightarrow y=\frac{180^{\circ}-120^{\circ}}{2}=30^{\circ}$
17. (B) Required simplified value

$$
\begin{aligned}
& \quad=\frac{2}{3} a b-\frac{5}{7} b c-\frac{2 a c}{3}-\frac{3}{2} b c+\frac{3}{5} a b+\frac{5}{2} c a \\
& =\left(\frac{2}{3} a b+\frac{3}{5} a b\right)+\left(-\frac{5}{7} b c-\frac{3}{2} b c\right)+\left(-\frac{2 a c}{3}+\frac{5 c a}{2}\right) \\
& =\left(\frac{10 a b+9 a b}{15}\right)+\left(\frac{-10 b c-21 b c}{14}\right)+\left(\frac{-4 a c+15 a c}{6}\right) \\
& \qquad\left(\frac{19 a b}{15}-\frac{31 b c}{14}+\frac{11 c a}{6}\right) \\
& \text { 18. (B) LHS }=\frac{0.000064-0.000027}{0.0016+0.0012+0.0009} \\
& \quad=\frac{0.000037}{0.0037}=0.01
\end{aligned}
$$

19. (A) Given $P R \| B C \Rightarrow \angle B=\angle B D P=74^{\circ}$

[ $\because$ Alternative angles]
Given $A B \| P Q$
i.e., $\mathrm{DB} \| \mathrm{PQ} \Rightarrow \angle \mathrm{BDP}+\angle \mathrm{P}=180^{\circ}$
$74+\angle \mathrm{P}=180^{\circ}$
$\angle \mathrm{P}=180^{\circ}-74^{\circ}=106^{\circ}$
20. (C) In $\triangle \mathrm{ABC}, \angle \mathrm{A}+\angle \mathrm{B}+\angle \mathrm{C}=180^{\circ}$
$\Rightarrow 3 \angle C+2 \angle C+\angle C=180$
$6 \angle \mathrm{C}=180^{\circ}$
$\angle \mathrm{C}=\frac{180^{\circ}}{6}=30^{\circ}$
$\therefore \quad \angle \mathrm{A}=3 \angle \mathrm{C}=90^{\circ}$
21. (D) Side of square $=\frac{48 \mathrm{~cm}}{4}=12 \mathrm{~cm}$

Area of square $=a^{2}=(12 \mathrm{~cm})^{2}=144 \mathrm{~cm}^{2}$
Given $\frac{1}{2} \times 2 x \times 48 \mathrm{~cm}^{2}=144 \mathrm{~cm}^{2}$
$x=\frac{144 \mathrm{~cm}^{2}}{48 \mathrm{~cm}}=3 \mathrm{~cm}$
22. (A) New radius $(R)=3 r$

New circumference
$=2 \pi R=2 \pi(3 r)=3(2 \pi r)=3 c$
23. (B) Let the number to be multiplied be $x$
$\therefore\left(\frac{7}{3}\right)^{-2} x=\left(\frac{5}{3}\right)^{-3}$
$\Rightarrow\left(\frac{3}{7}\right)^{2} x=\left(\frac{3}{5}\right)^{3}$
$\frac{9}{49} x=\frac{27}{125}$
$x=\frac{27^{3}}{125} \times \frac{49}{\not \varnothing_{1}}$
$=\frac{147}{125}$
24. (A) $\angle \mathrm{D}=\angle \mathrm{E} \quad[\because$ Corresponding angles $]$
$\therefore \angle \mathrm{D}=x=47^{\circ}$
In $\triangle \mathrm{ACD}, y+x+108^{\circ}=180^{\circ}$
$y+47^{\circ}+108^{\circ}=180^{\circ}$
$y=180^{\circ}-155^{\circ}=25^{\circ}$
25. (B) $2^{3^{64}}=2^{3^{2 \times 32}}=2^{\left(3^{2}\right)^{32}}=2^{3^{32}}$
26. (B) Let the first part be ₹ $x$
$\therefore \quad$ second part $=₹(45000-x)$
Given $10 \%$ ₹ $x=5 \%$ of ₹ $(45,000-x)$
$\Rightarrow \frac{10^{2}}{100} \times ₹ x=\frac{\nleftarrow}{100} \times ₹(45000-x)$
$2 x+x=₹ 45000$
$3 x=₹ 45000$
$x=\frac{₹ 45000}{3}=₹ 15,000$
$\therefore \quad$ Bigger part $=₹(45000-x)=₹ 30,000$
27. (B) Let $x \& y$ are supplimentary angles

Given $y=3 x$
But $x+y=180^{\circ}$
$x+3 x=180^{\circ}$
$4 x=180^{\circ}$
$x=\frac{180^{\circ}}{4}=45^{\circ}$
$y=3 x=3 \times 45^{\circ}=135^{\circ}$
28. (C) Given $C D \| A B \Rightarrow \angle D C E=\angle B=60^{\circ}$

In $\triangle \mathrm{ABC}, 55^{\circ}+60^{\circ}+\angle \mathrm{ACB}=180^{\circ}$
$\angle \mathrm{ACB}=180^{\circ}-115^{\circ}=65^{\circ}$
29. (D) Sum of other two angles

$$
=180^{\circ}-73^{\circ}=107^{\circ}
$$

30. (B) LCM of $3,4,6,12$ and 24 is 24

$$
\begin{aligned}
& \therefore-\frac{5}{6}=\frac{-20}{24},-\frac{3}{4}=-\frac{18}{24},-\frac{2}{3}=-\frac{16}{24},-\frac{17}{12}=-\frac{14}{24} \\
& \therefore-\frac{14}{24}>-\frac{16}{24}>-\frac{18}{24}>-\frac{19}{24}>-\frac{20}{24} \\
& \text { i.e., }-\frac{7}{12}>-\frac{2}{3}>-\frac{3}{4}>-\frac{19}{24}>-\frac{5}{6}
\end{aligned}
$$

## MATHEMATICS - 2 (MAQ)

31. ( $\mathrm{A}, \mathrm{B}, \mathrm{C}$ ) Let the smallest angle be ' $x$ '
$\therefore$ Sum of other twoangles $=180-x$
Given $x=\frac{1}{5}\left(180^{\circ}-x\right)$
$5 x=180^{\circ}-x$
$5 x+x=180^{\circ}$
$6 x=180^{\circ}$
smallest angle $(x)=\frac{180^{\circ}}{6}=30^{\circ}$
Required angles of a triangle are $30^{\circ}$, $45^{\circ}, 105^{\circ} ; 30^{\circ}, 65^{\circ}, 85^{\circ} ; 30^{\circ}, 50^{\circ}, 100^{\circ}$
32. (A, B, C, D)

Option (A) : $\left(y^{\frac{2}{3}}\right)^{9}=y^{\frac{2}{3} \times 9}=y^{6}$
Option (B) : $\left[\left(y^{6}\right)^{\frac{1}{2}}\right]^{2}=y^{6 \times \frac{1}{2} \times 2}=y^{6}$
Option (C) : $\left(y^{\frac{1}{3}}\right)^{18}=y^{\frac{1}{3} \times 18}=y^{6}$

Option (D) : $\frac{y^{18} \times y^{\frac{1}{2}}}{\left(y^{\frac{25}{2}}\right)}=y^{18} \times y^{\frac{1}{2}-\frac{25}{2}}=y^{6}$
33. (A, B, D) $\frac{1}{3}=0.33, \frac{1}{4}=0.25$
$\frac{7}{24}=0.29$ lies between 0.25 and 0.33 .
$\frac{13}{48}=0.27$ lies between 0.25 and 0.33 .
$\frac{8}{15}=0.53$ doesn't lie between 0.25 and 0.33
$\frac{5}{18}=0.27$ lie between 0.25 and 0.33 .
34. (A, B, C) A parallelogram has no line symmetry
35. (A, B, D) Option (A) is true of SAS congruency. Option (B) is true for ASA congruency. Option (D) is true because SAS congruency.

## REASONING

36. (C)

37. (C) $6^{3}=216$

$$
\begin{aligned}
& 15^{2}=225 \\
& 216=225
\end{aligned}
$$

38. (C)

39. (A)

40. (C) East, West, East, West

41. (C) Except is option (C) remaining options two letters are missing between in the letter series.
42. (D) First letter represent black circle and second letter represent white circles.
43. (C)

44. (A)

45. (C) Dictionary order is $2,1,4,5,3$

## CRITICAL THINKING

46. (B) Hence, number of people having one bicycle and 3 bicycle are equal. So, 29 families could be divided as
$29=10+10+9$
$29=14+14+1$
Or other groups as well as
Taking one group for example
$10 \times 3+10 \times 1+9 \times 2$
$30+10+18=58$
(OR)
$14 \times 3+14 \times 1+1 \times 2$
$42+14+2=58$
it is also possible with their pair.
Hence, 58 is the answer in each case.
47. (B) Since in the past the result was declared late by university. It has decided to conduct the examination in March/April in order to announce the result at proper times.
48. (B) Figures 1, 2 and 5 will form the square as shown in the following image;

49. (A) The given statements are as follows : Student A : D tore the book.

Student B: It was not me.
Student C: It was not E.
Student D: A is lying.
Student E:B is telling the truth
Let say A is saying the truth. Then, statement of B, C and E are also true which can't be possible as only three statements are true.
Let say $B, D$ and $E$ are telling the truth, this implies that E has torn the book.
50. (B) Wheel B moves anti-clockwise and slower than speed $P$ (rotation per second).


